Unit 02:

* BEVEL GEARS :-

- Bevel gears are used to transmit power between two intersecting shafts
- Spiral & straight bevel gears
- The elements of teeth of straight bevel gears are straight lines which converge into common apex pt
- The elements of teeth of spiral bevel gears are spiral curves which also converge into a common apex pt.
- Involute tooth profile for both.
- St. bevel gear - easy to design & mfg., create noise at high speed condition
- spiral bevel gear → difficult to design & costly to mfg, quiet operation at high speed, better strength.

- Bevel gear - classification on pitch angle.

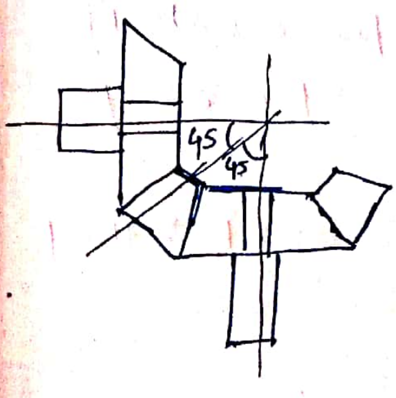
i) External bevel gear - pitch angle less than 90°

ii) Internal ~~[illegible]~~ bevel gear :- pitch angle more than 90.

iii) Crown bevel gear : pitch angle equal to 90°.

Special bevel gears :-

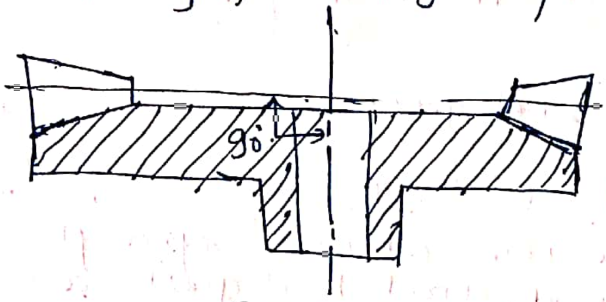
i) Miter Gear : Two identical gear mounted on shafts, which are intersecting at right angle.



→ pitch angle = 45° same.
- rotate at same speed
- dimensions same (Addendum, dedendum, pitch circle dia & module & teeth)
- shafts perpendicular to each other.

ii) crown gears :-

Gear having pitch angle equal to 90°

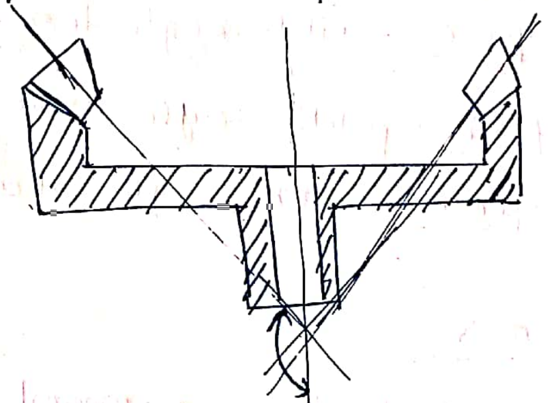


Crown Gear

- crown gear equivalent to rack in spur gearing
- bevel pinion & crown gear are always mounted on shafts, which are intersecting at angle more than 90°.

iii) Internal Bevel Gears :-

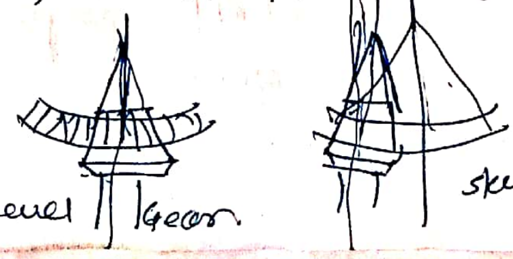
- pitch angle more than 90° &
- apex pt is on backside of teeth on that gear



- used in planetary gear box.

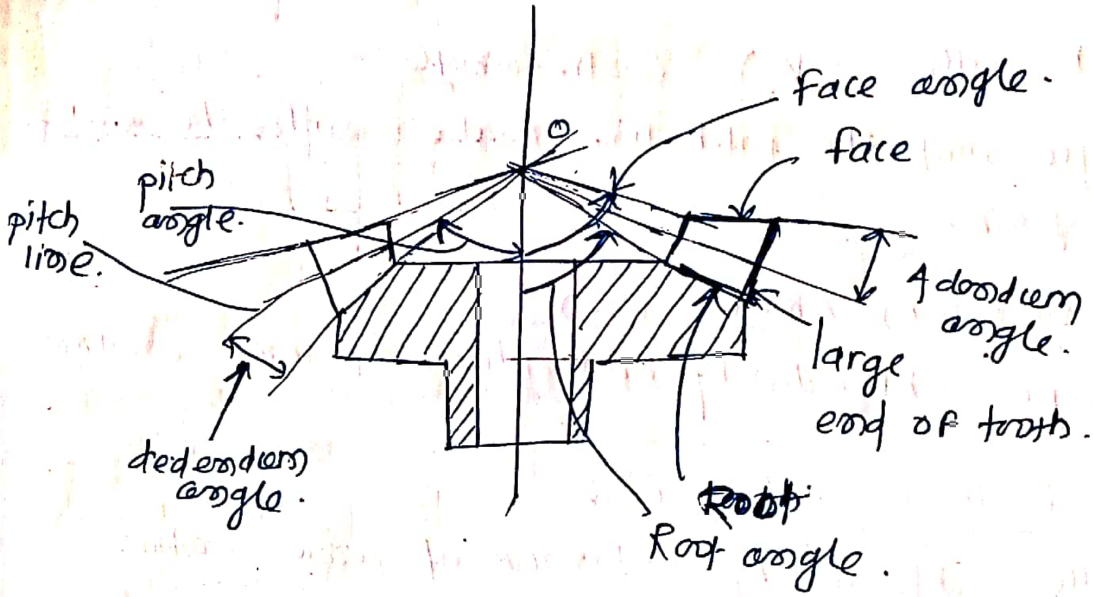
(iv) Skew Bevel gear :-

- When straight bevel gears are mounted on shafts which are non-parallel and non-intersecting, they are called skew bevel gear.
- apex pt. of pinion is offset with respect to that of gear.



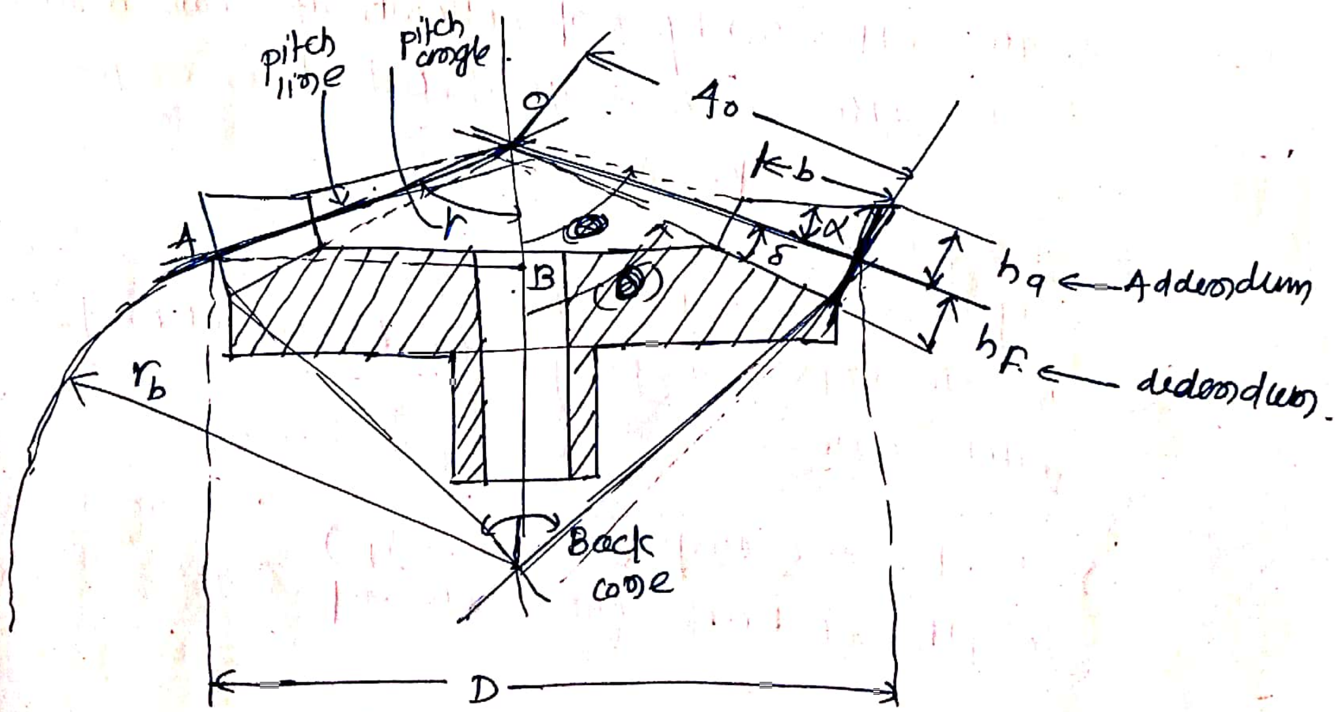
* Terminology of Bevel Gears:-

- Bevel gear is in the form of frustum of cone.



i) Pitch cone :- Pitch cone is an imaginary cone, the surface of which contains the pitch lines of all teeth in the bevel gear

ii) cone centre :- apex of pitch cone (O)



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iii) <u>cone Distance/pitch cone radius</u>

– length of pitch cone element (A_o)

iv) <u>pitch angle</u> :- (γ) centre angle

– angle that the pitch line makes with the axis of gear.

v) <u>Addendum Angle</u> :- (α)

– angle subtended by addendum at cone centre.

vi) <u>Dedendum angle</u> :- (δ)

– angle subtended by dedendum at cone centre.

vii) <u>Face angle</u> :-

– angle subtended by face of tooth at cone centre.

face angle = pitch angle + addendum angle.

$= (\gamma + \alpha)$

viii) <u>Root angle</u> :-

– Angle subtended by root of tooth at cone centre.

Root angle = pitch angle – dedendum angle

$= \gamma - \delta$.

ix) <u>Back cone</u> :-

– It is an imaginary cone

– its elements are perpendicular to elements of pitch cone.

x) <u>Back cone Distance</u> :- (r_b)

– length of back cone element.

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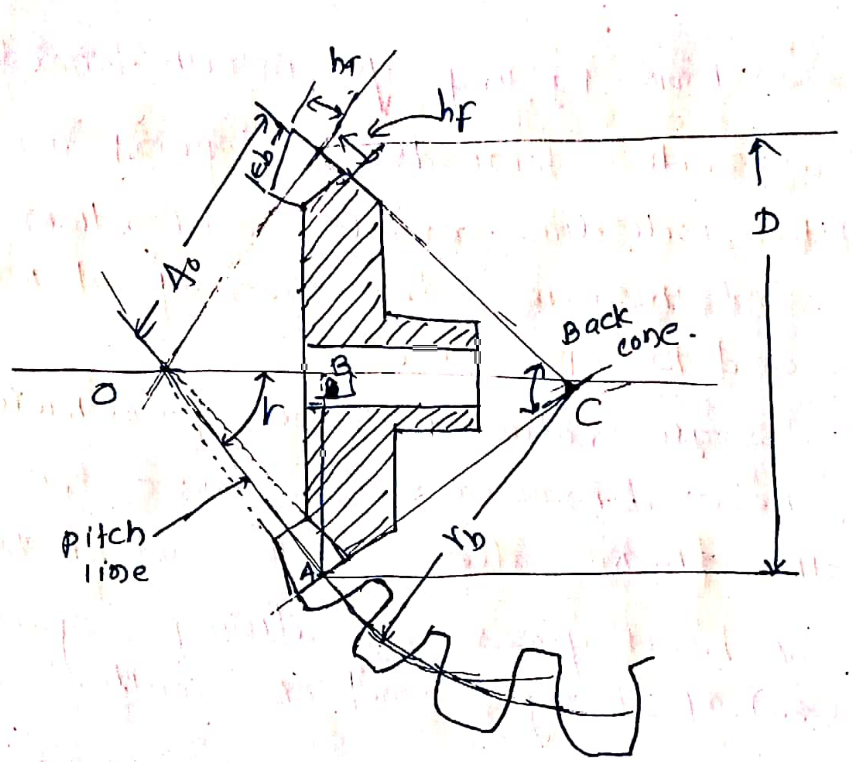
- It is observed from fig. that q's of tooth decreases in size as it approaches towards apex point O.
- Therefore pitch circle dia., module, addendum & dedendum ~~and there is~~ decreases as it moves from outer end to apex pt. O.
- There is no single value of these parameters.
- In practice, these dimensions are measured at largest tooth section called large end of tooth.
- Dimensions of bevel gears are always specified and measured at large end of tooth.

- Addendum (ha),
  dedendum (hf)
  pitch circle dia. (D)
  } specified at large end of tooth.

*



- figure shows an imaginary spur gear is considered in a plane perpendicular to tooth at the large end.

- r_b → pitch circle radius of imaginary spur gear &

  z' – no. of teeth on this gear

- formative or virtual spur gear.

$$2r_b = m \cdot z' \qquad z' = \frac{2r_b}{m} \quad \text{——— (1)}$$

m → module at large end of tooth.

z → actual no. of teeth on bevel gear

$$z = \frac{D}{m} \quad \text{——— (2)}.$$

from (1) & (2)

$$\frac{z'}{z} = \frac{2r_b}{D} \quad \text{——— (3)}$$

In fig. line AB ⊥ AC from Δ ABC

$$\sin(\angle BCA) = \frac{AB}{AC} \qquad \sin(90 - r) = \frac{(D/2)}{r_b}.$$

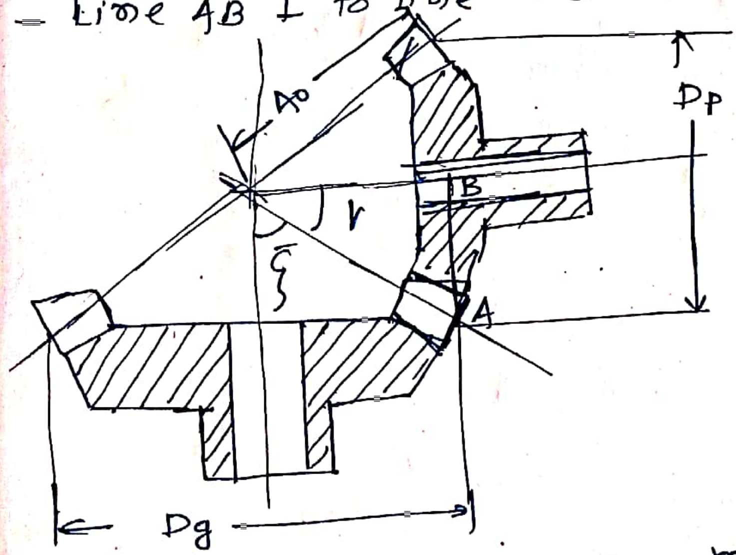
$$r_b = \frac{D}{2\cos\gamma} \quad \text{———} \quad (4)$$

substituting (4) in (3).

$$z' = \frac{z\,2r_b}{D} = \frac{z\,\cancel{2}\,\cancel{D}}{\cancel{D}\,\cancel{2}\cos\gamma} = \frac{z}{\cancel{}\cos\gamma}$$

$$\boxed{z' = \frac{z}{\cos\gamma}}$$

* consider a pair of bevel gear
- D_p & D_g → pitch circle dia. of pinion & gear respt.
  γ → pitch angle of pinion
  ξ → pitch angle of gear
- Line AB ⊥ to line OB. consider Δ OAB.



$$\tan\gamma = \frac{\overline{AB}}{\overline{OB}} = \frac{(D_p/2)}{(D_g/2)} = \frac{D_p}{D_g} = \frac{m Z_p}{m Z_g} = \frac{Z_p}{Z_g} \qquad \boxed{\tan\gamma = \frac{Z_p}{Z_g}}$$

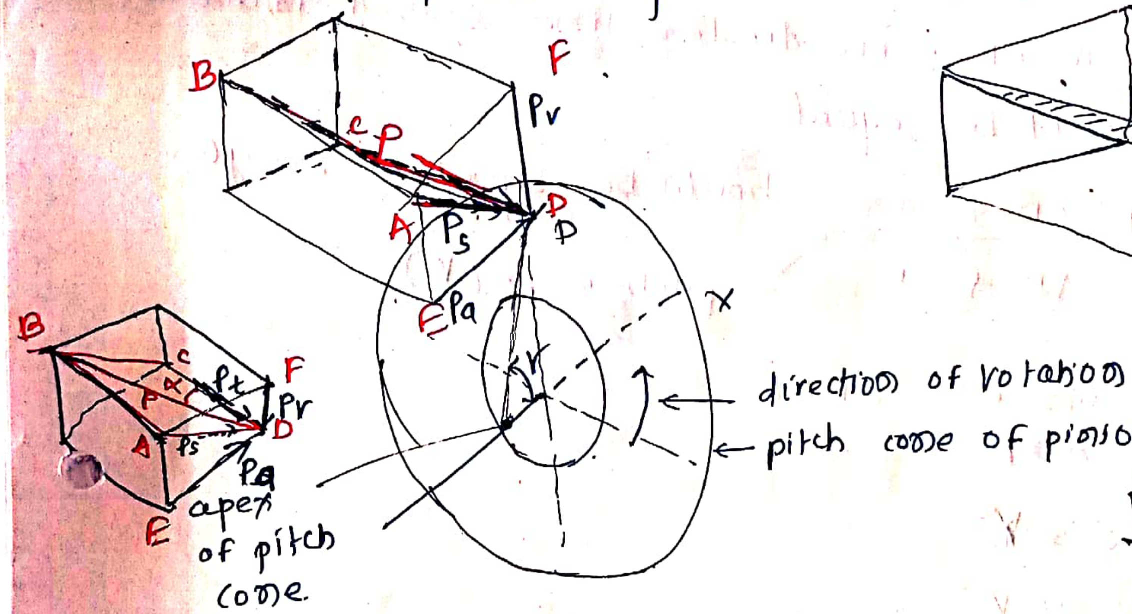
$$\boxed{\tan\xi = \frac{Z_g}{Z_p}} \qquad \gamma + \xi = \pi/2$$

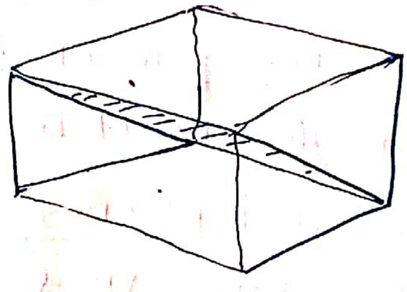
cone distance @ Ao

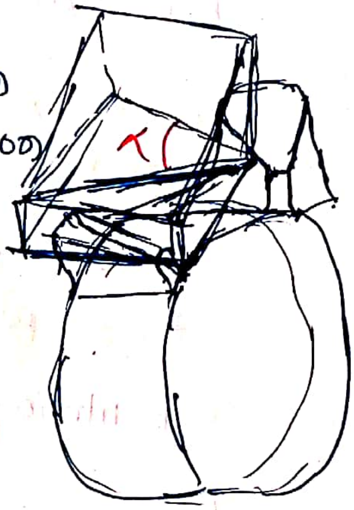
$$A_o = \overline{OA} = \sqrt{(\overline{AB})^2 + (\overline{BO})^2} = \sqrt{\left(\frac{D_p}{2}\right)^2 + \left(\frac{D_g}{2}\right)^2}$$

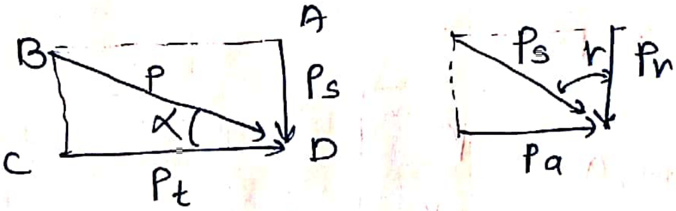
* Force Analysis :-

- In force analysis, it is assumed that force between two meshing teeth of a pair of bevel gears is concentrated at the midpoint along face width of tooth.









- Resultant force $P \rightarrow$ ~~dott~~ red line. act at midpoint D of face width of pinion.

- Resultant has three components

P_t = tangential component (N)

P_r = radical component (N)

P_a = axial component (N)

→ consider plane ABCD

Δ BCD

$$\tan\alpha = \frac{BC}{CD} = \frac{P_s}{P_t}$$

$$P_s = P_t \tan\alpha$$

P_s = separating component

α = pressure angle.

- Separating force P_s is perpendicular to pitch line OD.

$$AD \perp OD$$

- line FD is vertical, while line OX is horizontal

$$FD \perp OX$$

- so two pairs of perpendicular line & their included angle should be equal.
- angle betn OD & OX should be equal to angle betn line AD & FD. → pitch angle (γ)

→ Consider plane DEAF.
from Δ ADF

$$P_r = P_s \cos \gamma$$

$$P_a = P_s \sin \gamma$$

substituting P_s

$$P_r = P_t \tan \alpha \cdot \cos \gamma$$

$$P_a = P_t \tan \alpha \cdot \sin \gamma$$

$$P_t = \frac{M_t}{r_m}$$

M_t = torque transmitted by gear N mm

r_m = radius of pinion at mid point along face width (mm)

$$r_m = \overline{AC} - \overline{AB}$$

$$\overline{AC} = \frac{D_p}{2}$$

$$\sin \gamma = \frac{\overline{AB}}{(b/2)} \qquad \overline{AB} = \frac{b}{2} \sin \gamma$$

$$\boxed{r_m = \frac{D_p}{2} - \frac{b \sin \gamma}{2}}$$

b = face width of tooth (mm)

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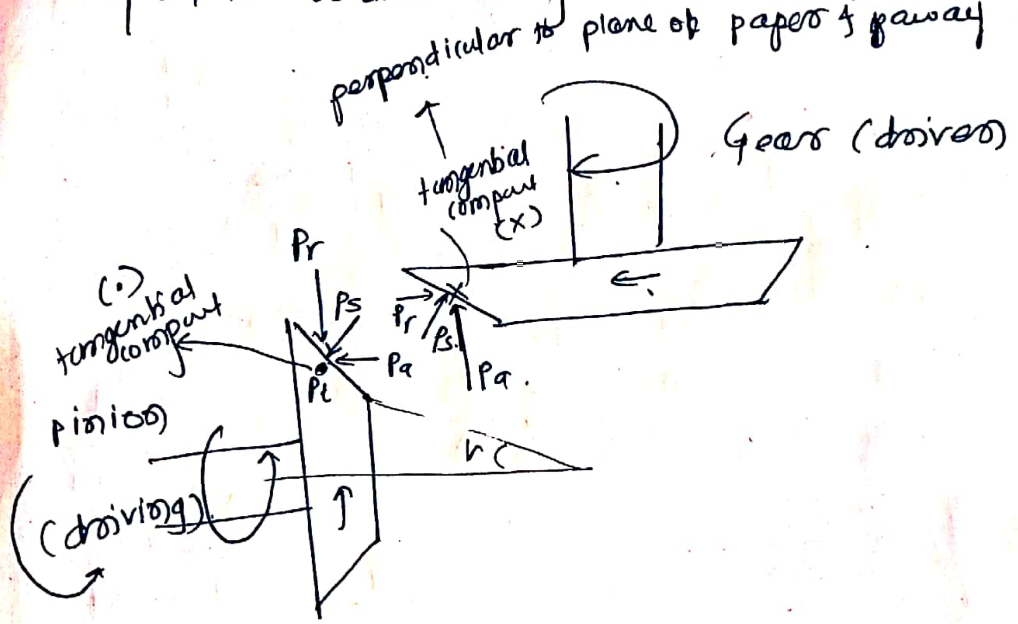
- The radial component on gear is equal to axial component on pinion. similarly axial component on gear is equal to radial component on pinion.



Direction :-

i) Tangential component (P_t)

- direction of tangential component for driving gear is opposite to direcⁿ of rotation

- direction of P_t for driven is in the direcⁿ of rotation.

ii) Radial (P_r)

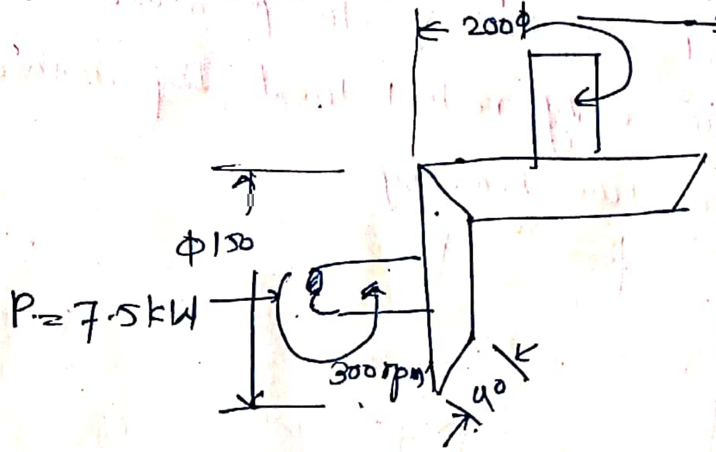
- on pinion act towards centre of pinion
- on gear act towards centre of gear

iii) Thrust (P_a)

- Thrust component on pinion is equal & opposite of radial component on gear & viceversa.

Example :-

A pair of bevel gears transmitting 7.5 kW at 300 rpm is shown in fig. The pressure angle is 20°. Determine the components of the resultant gear tooth force & draw a FBD of forces acting on pinion & gear.



$$\rightarrow M_t = \frac{60 \times 10^6 \,(P\,kW)}{2\pi n_p} = 238.732 \text{ kN mm}$$

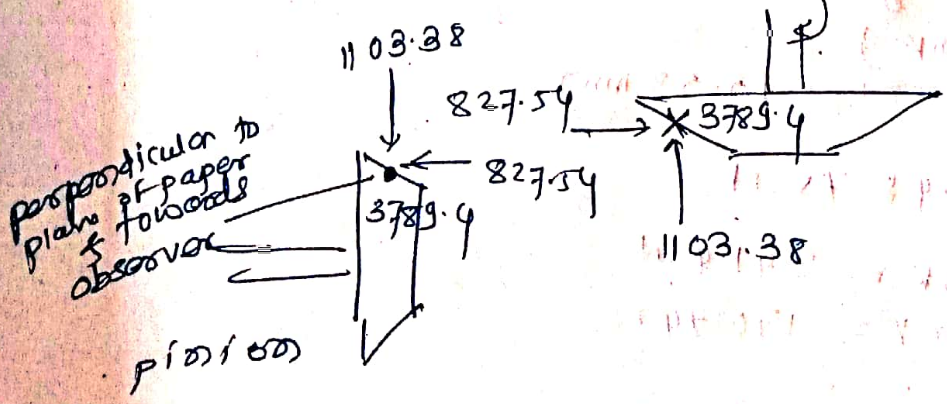
$$\tan \gamma = \frac{Z_p}{Z_g} = \frac{D_p}{D_g} = \frac{150}{200} \qquad \gamma = 36.87^\circ$$

$$r_m = \left[\frac{D_p}{2} - \frac{b \sin \gamma}{2}\right] = 63 \text{ mm}$$

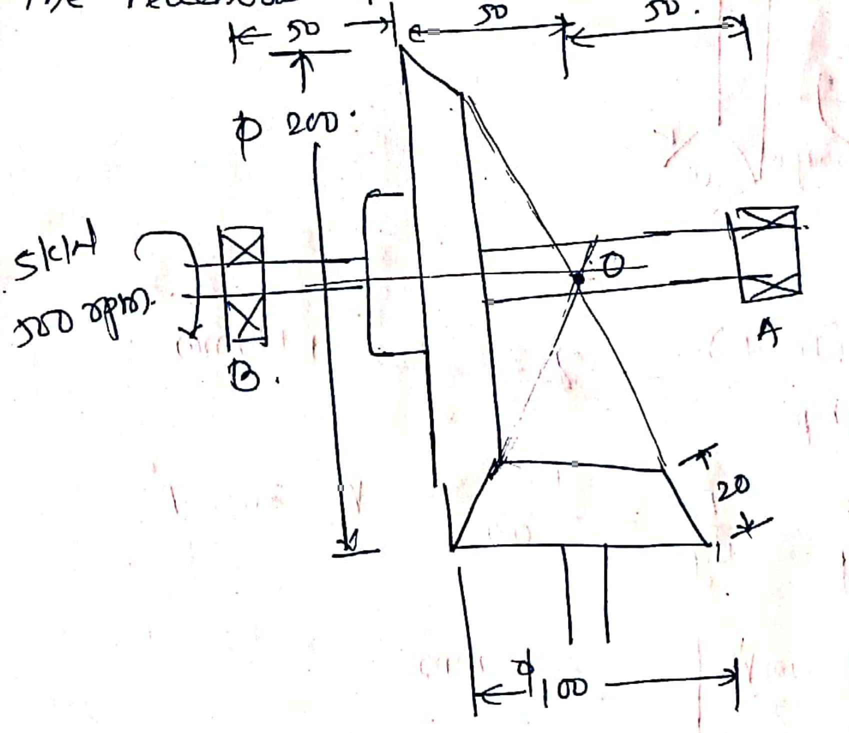
$$P_t = \frac{M_t}{r_m} = 3789.40 \text{ N}$$

$$P_r = P_t \tan \alpha \cos \gamma = 1103.38 \text{ N}$$

$$P_a = P_t \tan \alpha \sin \gamma = 827.54 \text{ N.}$$



② The dimensions of a pair of bevel gears are given in fig. The gear G delivers 5 kW of power at 500 rpm to the output shaft. The bearings A & B are mounted on output shaft in such a way that the bearing B can take radial as well as thrust load, while the bearing A can take only radial load. Determine the reactions at the two bearings



$$\frac{n_p}{n_g} = \frac{D_g}{D_p} \qquad n_p = 1000 \text{ rpm}$$

$$M_t = \frac{60 \times 10^6 (P_{kW})}{2\pi n_p} = 47.746 \text{ Nmm.}$$

$$\tan \gamma = \frac{Z_p}{Z_g} = \frac{D_p}{D_g} = 0.5 \qquad \gamma = 26.57$$

$$r_m = \frac{(D_p - b \sin\gamma)}{2} = 45.53 \text{ mm}$$

$$P_t = \frac{M_t}{r_m} = 1048.75 \text{ N}$$

$$P_r = P_t \tan\alpha \cdot \cos\gamma = 341.4 \text{ N}$$

$$P_a = P_t \tan\alpha \sin\gamma = 170.74 \text{ N.}$$

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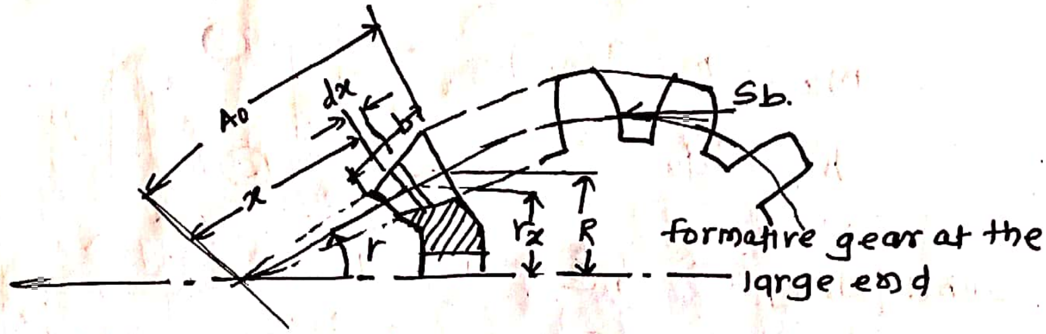
* Beam strength of Bevel Gears :-

- size of c/s of tooth of bevel gear varies along face width

- In order to determine the beam strength of the tooth of a bevel gear, it is considered to be a equivalent to a formative spur gear in a plane perpendicular to the tooth element.



Beam strength of Bevel tooth .

- consider an elemental section of the tooth at a distance 'x' from the apex o and having a width 'dx'.

- Applying lewis equation to a formative spur gear at a distance x from pex.

$$\delta(S_b) = m_x \cdot b_x \sigma_b \cdot Y \qquad \text{—— ①}$$

$\delta(S_b)$ = beam strength of elemental section (N)
m_x = module of the tooth at section x : (mm)
b_x = face width of elemental section (mm)
Y = Lewis form factor based on virtual no. of teeth.

- From fig.

$$\frac{r_x}{x} = \frac{R}{A_0} \qquad r_x = \left(\frac{x R}{A_0}\right) \qquad \text{—— ②}$$

- Module of tooth at elemental section

$$m_x = \frac{2r_x}{z} = \frac{2}{z}\left(\frac{xR}{A_0}\right) = \frac{2xR}{zA_0} \quad \text{--- } ③$$

- At large end of tooth

$$m = \frac{2R}{z} \quad \text{--- } ④$$

substituting ④ in ③

$$m_x = m(x/A_0) \quad \text{--- } ⑤$$

$$b_x = dx \quad \text{--- } ⑥$$

Substituting ⑤ & ⑥ in ①

$$\delta(S_b) = m\left(\frac{x}{A_0}\right) \times dx \times \sigma_b \cdot Y$$

$$\delta(S_b) = \frac{m\,\sigma_b \cdot x \cdot dx \cdot Y}{A_0}$$

Multiplying both side by r_x

$$\delta(S_b) \cdot r_x = \frac{m \cdot \sigma_b \cdot x \cdot dx \cdot Y}{A_0} \times r_x = \frac{m\,\sigma_b \cdot x \cdot dx \cdot Y}{A_0} \times \left(\frac{xR}{A_0}\right)$$

$$\delta(S_b) r_x = \frac{m\,\sigma_b \cdot x^2 R \cdot Y\, dx}{A_0^2}$$

Integrating on both side

$$\int \delta(S_b) r_x = \frac{m\,\sigma_b \cdot YR}{A_0^2} \int x^2 dx$$

The lefthand side indicates the torque (M_t)

M_t = tangential force × radius $\int \delta(S_b) \cdot r_x$.

$$M_t = \left(\frac{m\,\sigma_b \cdot YR}{A_0^2}\right) \int_{\text{inner radius} = A_0 - b}^{\text{outer radius} = A_0} x^2 dx$$

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$$M_t = \left(\frac{m\,\sigma_b \cdot Y \cdot R}{A_0^2}\right) \left[\frac{x^3}{3}\right]_{(A_0-b)}^{A_0}$$

$$M_t = \frac{m \cdot \sigma_b \cdot YR}{A_0^2} \frac{1}{3}\left[A_0^3 - (A_0-b)^3\right]$$

$$= \frac{m\,\sigma_b \cdot YR}{A_0^2} \frac{1}{3}\left[A_0^3 - \left(A_0^3 - 3A_0^2 b + 3A_0 b^2 - b^3\right)\right]$$

$$= \frac{m\,\sigma_b \cdot YR}{A_0^2} \frac{1}{3}\left[\cancel{A_0^3} - \cancel{A_0^3} + 3A_0^2 b - 3A_0 b^2 + b^3\right]$$

$$M_t = m\,\sigma_b \cdot YR\,b \cdot \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right]$$

– Beam strength (S_b) as a tangential force at large end of tooth

$$M_t = \left[m \cdot \sigma_b \cdot Y \cdot b \left(1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right)\right] \times R$$

$$M_t = S_b \cdot R$$

$$\boxed{S_b = m \cdot \sigma_b \cdot Y b \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right]}$$

– face width of bevel gear is limited to (1/3) of cone distance. so last term of bracket will never be more than (1/27). It is so small. so

Neglecting last term

$$\boxed{S_b = m \cdot b \cdot \sigma_b \cdot Y \left(1 - b/A_0\right)}$$

Lewis eqⁿ for bevel Gear

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$$M_t = \left(\frac{m \sigma_b \cdot Y \cdot R}{A_0^2}\right) \left[\frac{x^3}{3}\right]_{(A_0-b)}^{A_0}$$

$$M_t = \frac{m \cdot \sigma_b \cdot Y R}{A_0^2} \frac{1}{3}\left[A_0^3 - (A_0-b)^3\right]$$

$$= \frac{m \sigma_b \cdot Y R}{A_0^2} \frac{1}{3}\left[A_0^3 - \left(A_0^3 - 3A_0^2 b + 3A_0 b^2 - b^3\right)\right]$$

$$= \frac{m \sigma_b \cdot Y R}{A_0^2} \frac{1}{3}\left[\cancel{A_0^3} - \cancel{A_0^3} + 3A_0^2 b - 3A_0 b^2 + b^3\right]$$

$$M_t = m \sigma_b \cdot Y R b \cdot \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right]$$

- Beam strength (S_b) as a tangential force at large end of tooth

$$M_t = \left[m \cdot \sigma_b \cdot Y \cdot b \left(1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right)\right] \times R$$

$$M_t = S_b \cdot R$$

$$\boxed{S_b = m \cdot \sigma_b \cdot Y b \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right]}$$

- face width of bevel gear is limited to (1/3) of cone distance. so last term of bracket will never be more than (1/27). It is so small. so Neglecting last term

$$\boxed{S_b = m \cdot b \cdot \sigma_b \cdot Y \left(1 - b/A_0\right)}$$

Lewis's eqn for bevel Gear

S_b = beam strength of tooth (N)
m = module at large end of tooth
b = Face width.

σ_b = permissible bending stress $(S_{ut}/3)$ (N/mm^2)
Y = Lewis form factor based on formative no. of teeth.
A_0 = cone distance (mm)

$[1 - b/A_0)$ = bevel factor.

$$\boxed{S_b = M_t/R}$$

- Beam strength indicates the maximum value of the tangential force at large end of tooth that the tooth can transmit without bending failure.

$P_t = 2M_t/D$.

- Beam strength should always more than effective force between the meshing teeth at the large end of the tooth.

- Face width of bevel gear is generally taken as 10m or $(A_0/3)$ whichever is small.

* Wear strength of Bevel Gears :-

- The contact between the two meshing teeth of straight bevel gear is line contact, which is same as spur gear
- Bevel gear is considered equivalent to a formative spur gear in a plane which is perpendicular to the tooth at the large end.
- Applying Buckingham's eq^n to these formative gears,

$$S_w = b\,Q\,d_p'\,K \quad \text{———— (1)}$$

b = face width (mm)
Q = ratio factor
d_p' = pitch circle dia. of formative pinion. (mm)
K = material const. (N/mm^2)

$$d_p' = 2r_b \quad \& \quad d_p' = \frac{D_p}{\cos V}$$

D_p = pitch circle dia. of pinion at large end of teeth.

$$S_w = \frac{b\,Q \cdot D_p \cdot K}{\cos V} \quad \text{———— (2)}$$

- In case of bevel gears, either pinion or gear is gradually overhanging.
- It is subjected to deflection under the action of tooth forces & it has been found that to transmit the load, only three quarter of face width is effective.
- Modifying eq^n (2) gives

$$S_w = \frac{0.75\,b\,Q \cdot D_p\,K}{\cos V}$$

- Above eqn is derived for formative pair of pinion & gear.

Ratio factor Q is

$$Q = \frac{2 Z_g'}{Z_g' + Z_p'} =$$

$$Z_p' = \frac{Z_p}{\cos\gamma} \quad \& \quad Z_g' = \frac{Z_g}{\cos\beta} = \frac{Z_g}{\cos(90-\gamma)} = \frac{Z_g}{\sin\gamma}$$

$$Q = \frac{2(Z_g/\sin\gamma)}{\frac{Z_g}{\sin\gamma} + \frac{Z_p}{\cancel{\sin\gamma}\cos\gamma}} = \frac{2 Z_g}{Z_g + Z_p \tan\gamma}$$

- The material constant k is same for spur gear & given by

$$K = \frac{\sigma_c^2 \sin\alpha \cdot \cos\alpha \left[\frac{1}{E_p} + \frac{1}{E_g}\right]}{1.4}$$

- When both pinion & gear are made of steel & press. angle is 20°, the value of K is

$$K = 0.16\left(\frac{BHN}{100}\right)^2$$

- The wear strength (Sw) indicates the maximum value of tangential force at the large end of the tooth that the tooth can transmit without pitting failure.

- $S_w > P_{eff}$

* Effective load on Gear teeth :-

— The tangential component due to power transmission, considered to be acting at the large end of tooth, is determined using following two eqns.

$$M_t = \frac{60 \times 10^6 \, (P_{kW})}{2\pi n_p}$$

$$P_t = \frac{2M_t}{D}$$

— In adition to tangential component due to power transmission, there is the dynamic load.

— <u>Dynamic load</u> :- approximate estimation by means of the velocity factor in preliminary stages of gear design

$$P_{eff} = \frac{C_s}{C_v} \cdot P_t$$

C_s = service factor

C_v = velocity factor

$$C_v = \frac{6}{6+V} \rightarrow \text{for cut teeth}$$

$$C_v = \frac{5.6}{5.6+\sqrt{V}} \rightarrow \text{for generated teeth.}$$

V = pitch line velocity in m/s

— <u>Dynamic load</u> : Buckingham's eqn in final stage of gear design

$$P_{eff} = C_s \cdot P_t + P_d$$

$$P_d = \frac{21V\,(Ceb + P_t)}{21V + \sqrt{Ceb + P_t}}$$

P_d = incremental dynamic load N
v = pitch line velocity (m/s)
C = Deformation factor N/mm²
e = sum of errors bet^n two meshing teeth (mm)
b = face width
P_t = tangential force

Bevel gears made of steel $\Rightarrow C = 11400$ N/mm²

$$S_w = P_{eff} \cdot (FS)$$

Example

① A pair of bevel gears, with 20° pressure angle, consists of a 20 teeth pinion meshing with a 30 teeth gear. The module is 4 mm while the face width is 20 mm. The material for the pinion and gear is steel 50C4 ($S_{ut} = 750\ N/mm^2$). The gear teeth are lapped & ground (Class-3) & the surface hardness is 400 BHN. The pinion rotates at 500 rpm and receives 2.5 kW power from the electric motor. The starting torque of the motor is 150% of the rated torque. Determine the factor of safety against bending failure and against pitting failure.

Both pinion & gears are made of same material so pinion is weaker.

$$\tan\gamma = \frac{Z_p}{Z_g} = \frac{20}{30} \qquad \gamma = 33.69°$$

$$Z_p' = \frac{Z_p}{\cos\gamma} = 24.04$$

$$Y = 0.3371$$

$$\sigma_b = \frac{S_{ut}}{3} = 250\ N/mm^2$$

$$D_p = m Z_p = 80\ mm$$

$$D_g = m Z_g = 120\ mm$$

$$A_0 = \sqrt{(D_p/2)^2 + (D_g/2)^2} = 72.11\ mm$$

$$S_b = m b \cdot \sigma_b \cdot Y\,(1 - b/A_0) = 4872.37\ N$$

Wear :-

$$Q = \frac{2 Z_g}{Z_g + Z_p \cdot \tan\gamma} = 1.385$$

$$K = 0.16\,(BHN/100)^2 = 2.56\ N/mm^2$$

$$S_w = \frac{0.75\, b\, Q\, D_p\, K}{\cos\gamma} = 5113.53\ N.$$

Tangential force

$$M_t = \frac{60 \times 10^6 \; P_{kW}}{2\pi n_p} = 47746.48 \; Nmm$$

$$P_t = \frac{2M_t}{D_p} = 1193.66 \; N.$$

Effective load based on Buckingham's eqn

$$v = \frac{\pi D_p n_p}{60 \times 10^3} = 2.094 \; m/s$$

$C = 11400 \; N/mm^2$

$b = 20 \; mm$

$P_t = 1193.66 \; N$

$$P_d = \frac{21V\,(Ceb + P_t)}{21V + \sqrt{Ceb + P_t}} = 1653.25 \; N.$$

Effective load

$$P_{eff} = C_s \cdot P_t + P_d = 3443.74 \; N$$

<u>Factor of safety</u> :.

<u>Bending</u>

$$fs = \frac{S_b}{P_{eff}} = 1.41$$

<u>Wear</u>

$$fs = \frac{S_w}{P_{eff}} = 1.48$$

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Example

② A pair of straight bevel gears mounted on shafts which are intersecting at right angles, consists of a 24 teeth pinion meshing with a 32 teeth gear. The pinion shaft is connected to an electric motor developing 12.5 kW rated power at 1440 rpm. The starting torque of the motor is 150 %. of the rated torque. The pressure angle is 20°. Both gears are made of case hardened steel (S_{ut} = 750 N/mm^2). The teeth on gears are generated and finished by grinding and lapping processes to meet the requirements of Class-3 Grade. The factor of safety in preliminary stages of gear design is 2.

i) In initial stages of gear design, assume that the velocity factor accounts for the dynamic load & that the pitch line velocity is 7.5 m/s. Estimate the module based on beam strength.

ii) Select the first preference value of the module and calculate the main dimensions of the gears.

iii) Determine the dynamic load using Buckingham's equation and find out the effective load for the above dimensions. What is the correct factor of safety for bending?

iv) Specify the surface hardness for the gears assuming the factor of safety of 2 for wear consideration.

Example

(3) A pair of straight bevel gear is mounted on shafts, which are intersecting at right angles. The no. of teeth on the pinion and gear are 21 & 28 resp. The pressure angle is 20°. The pinion shaft is connected to an electric motor developing 5 kW rated power at 1440 rpm. The service factor can be taken as 1.5. The pinion & gear are made of steel (S_{ut} = 750 N/mm²) and heat-treated to a surface hardness of 380 BHN. The gears are machined by a manufacturing process, which limit the error between the meshing teeth to 10 μm. The module & face width are 4 mm & 20 mm respectively.

Determine the factor of safety against bending as well as against pitting failure.

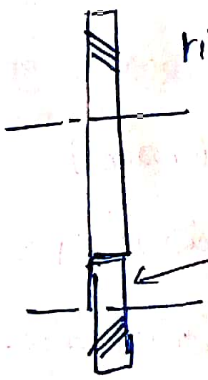
Unit 02 Helical Gear

Helical Gears :-

- Teeth of the gear cut in the form of a helix on a pitch cylinder.
- In helical gears, the contact betn meshing teeth begins with a point on the leading edge of the tooth and gradually extends along diagonal line across the tooth.
- gradual pick-up of load by teeth resulting in smooth engagement & quite operation at high speed.

- **Parallel Helical Gear** :- operate on two parallel shafts.
- magnitude of helix angle is same for pinion & gear but hand of helix is opposite.
- Right hand pinion meshes with left hand gear & vice-versa.

- **Crossed Helical Gear** :- mounted on shafts with crossed axes.
- same or opposite hand of helix.



right hand helical teeth — lines are slopping downward to the right side

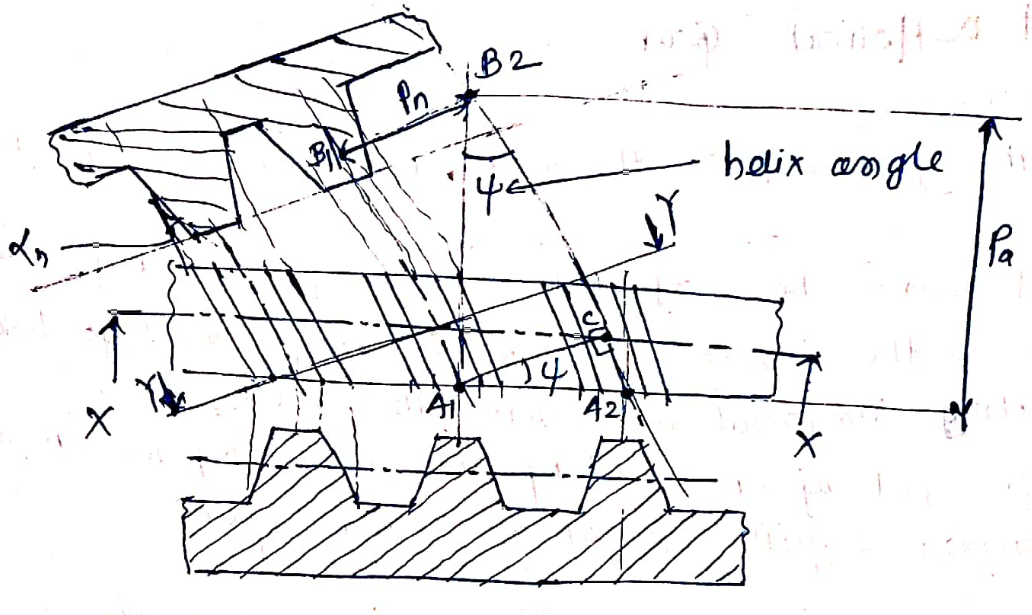
left hand helical teeth — lines are slopping upward to right hand side.

circular pitch $p = \frac{\pi d}{z}$

Module $= m = \frac{d}{z} = \frac{1}{P}$

Diametral pitch a $P = \frac{z}{d}$

$p \cdot P = \pi$



Section at X-X

$\overline{A_1B_1}$ & $\overline{A_2B_2}$ → centre lines of adjacent teeth taken on pitch plane.

$A_1B_2A_2 = \psi$ = helix angle

– angle betⁿ axis of shaft and the centre line of tooth taken on pitch plane.

X-X → plane of rotation

YY → plane perpendicular to tooth element

(p) $\overline{A_1A_2}$ → transverse circular pitch measured in plane of rotation

(p_n) $\overline{A_1C}$ → normal circular pitch measured in a plane perpendicular to tooth element

Module

From ΔA_1A_2C

$$\cos\psi = \frac{p_n}{p} = \frac{\overline{A_1C}}{A_1A_2}$$

$$\boxed{p_n = p\cos\psi}$$

substituting $pP = \pi$ in above expression

$pP = \pi \Rightarrow p = \frac{\pi}{P}$

$p_n P_n = \pi \Rightarrow p_n = \frac{\pi}{P_n}$

$$\frac{\pi}{P_n} = \frac{\pi}{P}\cos\psi$$

$$\boxed{P_n = \frac{P}{\cos\psi}}$$

P_n = normal diametral pitch

P = transverse diametral pitch.

Substituting $P = \frac{1}{m}$ $P_n = \frac{1}{m_n}$ in above eqn.

$$\frac{1}{m_n} = \frac{(1/m)}{\cos\psi}$$

$$\boxed{m_n = m\cos\psi}$$

m_n = normal module (mm)
m = transvese module (mm)

Distance $\overline{A_1B_2}$ = axial pitch (P_a)

From $\Delta\ A_1A_2B_2$

$$\tan\psi = \frac{\cancel{A_1C}}{\cancel{B_2C}} = \frac{A_1A_2}{A_1B_2} = \frac{p}{P_a}$$

$$P_a = \frac{p}{\tan\psi}$$

Pressure Angle

- There are two pressure angles, transvese pressure angle 'α' and normal pressure angle 'α_n' in their respective plane.
- They are related by following expression

$$\cos\psi = \frac{\tan\alpha_n}{\tan\alpha}$$

Pitch circle Diameter:-

→ normal pressure angle usually = 20°.
pitch circle dia. d" of helical gear is given by

$P = \frac{\pi d}{z}$

$P \times p = \pi$

$$\therefore\ d = \frac{zp}{\pi} = \frac{z}{P} = zm = \frac{zm_n}{\cos\psi}$$

$$\boxed{d = \frac{zm_n}{\cos\psi}}$$

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Centre to centre Distance :-

The centre to centre distance 'a' between the two helical gears having Z_1 & Z_2 as teeth no. is given by

$$a = \frac{d_1}{2} + \frac{d_2}{2} = \frac{Z_1 m_n}{2\cos\psi} + \frac{Z_2 m_n}{2\cos\psi}$$

$$\boxed{a = \frac{m_n(Z_1 + Z_2)}{2\cos\psi}}$$

* Speed ratio for helical gear

$$i = \frac{\omega_p}{\omega_g} = \frac{Z_g}{Z_p}.$$

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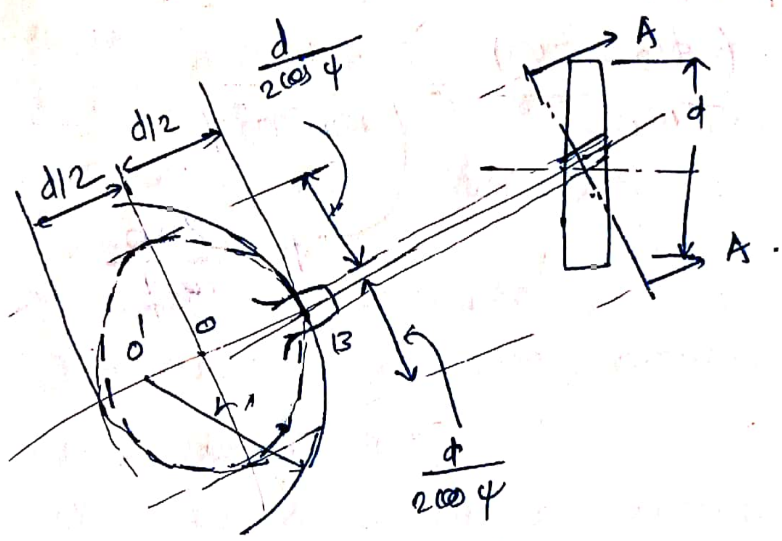
* Virtual No. of teeth :-



section at A-A.

- pitch cylinder of helical gear is cut by plane A-A which is normal to tooth element
- intersection of plane A-A and pitch cylinder produces ellipse. (shown by dotted line)

→ semi major axis $= \dfrac{d}{2\cos\psi} = a$

semi minor axis $= d/2 = b$

- Analytical geometry → radius of curvature (r')

$$r' = (a^2/b)$$

$$r' = \frac{d}{2\cos^2\psi}$$

→ In design of helical gears, an imaginary spur gear is considered in plane A-A with centre at 'O' having pitch circle radius r' and module m_n.

- It is called formative or virtual of spur gear.
- pitch circle dia. of virtual gear

= d' = pitch circle dia.

$$d' = 2r' = \frac{d}{\cos^2\psi}$$

The no. of teeth z' on this imaginary spur gear is called virtual no. of teeth.

$$z' = \frac{2\pi r'}{p_n} = \frac{2\pi\left(d/2\cos^2\psi\right)}{\pi m_n} = \frac{d}{m_n\cos^2\psi}$$

$$d = \frac{z m_n}{\cos\psi}$$

$$\boxed{z' = \frac{z}{\cos^3\psi}}$$

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Example :-

A pair of parallel helical gears consists of a 20 teeth pinion meshing with a 40 teeth gear. the helix angle is 25° and the normal pressure angle is 20°. the normal module is 3 mm.

Calculate

i) Transverse module

ii) transverse pressure angle

iii) Axial pitch.

iv) pitch circle dia. of pinion & gear

v) centre distance

vi) Addendum & dedendum circle dia. of pinion.

$$\rightarrow \quad m = \frac{m_n}{\cos\psi}$$

$$\tan\alpha = \frac{\tan\alpha_n}{\cos\psi}$$

$$P_a = \frac{P}{\tan\psi}$$

$$d_p = \frac{Z_p \cdot m_n}{\cos\psi}$$

$$d_g = \frac{Z_g \cdot m_n}{\cos\psi}$$

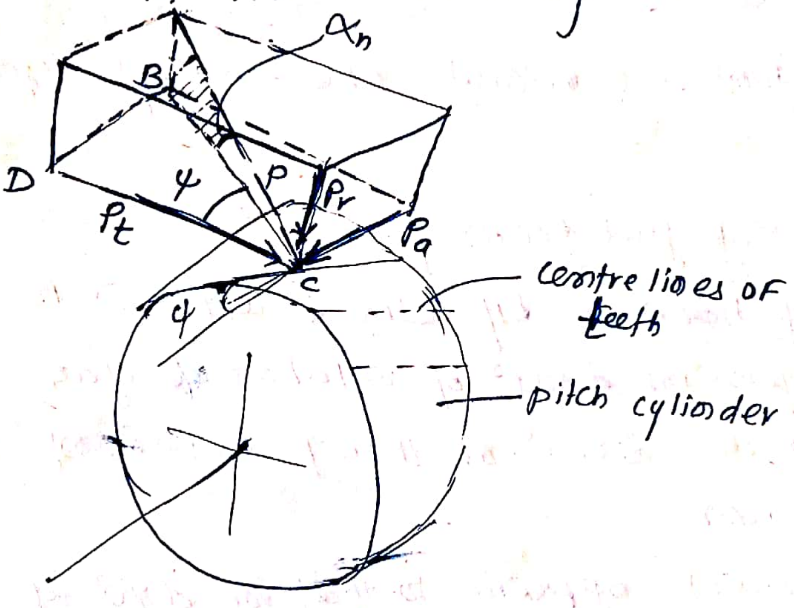
$$a = \frac{d_p + d_g}{2}$$

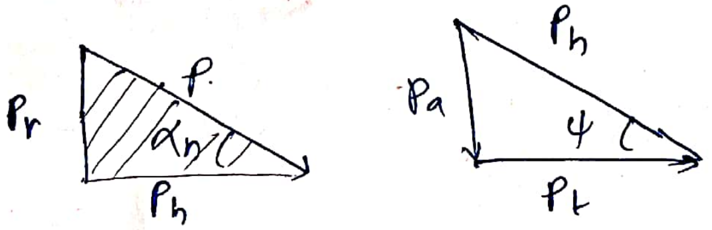
$$d_a = m_n\left[\frac{Z}{\cos\psi} + 2\right]$$

$$d_f = m_n\left[\frac{Z}{\cos\psi} - 2.5\right]$$

* Force Analysis :-

Resultant force P acting on the tooth of a helical gear is resolved into three components P_t, P_r and P_a.





$\alpha_n \Rightarrow$ normal press. angle in plane ABC

ψ = helix angle in plane BCD

From Δ ABC $\quad P_r = P \sin \alpha_n$ ——— (a)

$\overline{BC} = P \cos \alpha_n$ ——— (b)

from Δ BDC

$P_a = \overline{BC} \sin \psi = P \cos \alpha_n \sin \psi$ ——— (c)

$P_t = \overline{BC} \cos \psi = P \cos \alpha_n \cos \psi$ ——— (d)

From (c) & (d)

$$\boxed{P_a = P_t \tan \psi}$$

from (a) & (d)

$$\boxed{P_r = P_t \left(\frac{\tan \alpha_n}{\cos \psi} \right)}$$

$$P_t = \frac{2M_t}{d} \qquad M_t = \frac{60\times10^6\ P_{kW}}{2\pi n_p}.$$

Direction :- Tangential & radial same as spur gear

Thrust -

i) select driving gear from pair

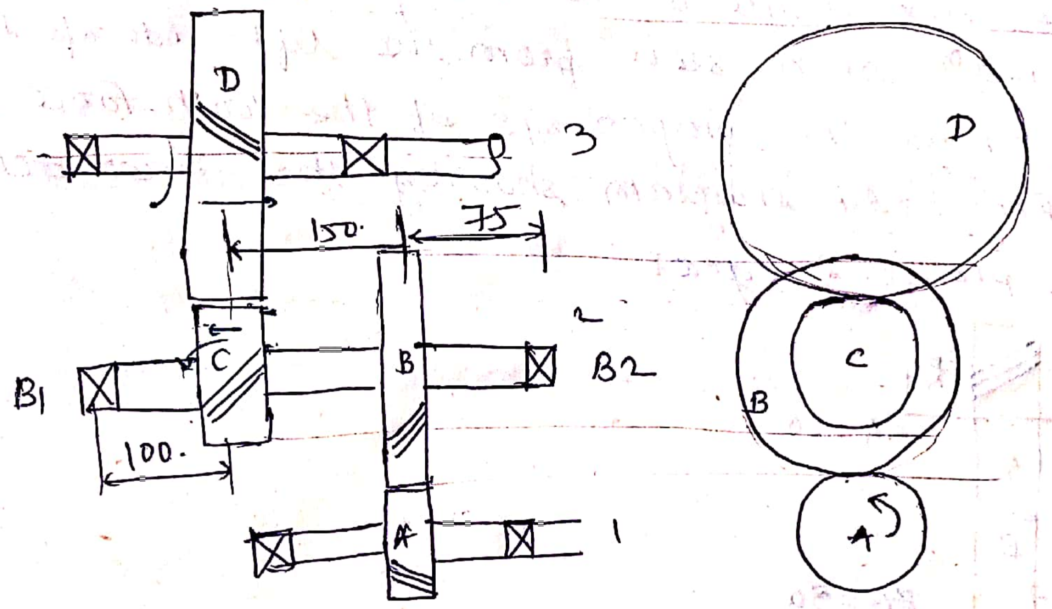
ii) use right hand for RH-helix & left hand for LH-helix.

iii) Keep fingers in direcⁿ of rotation of gear & thumb will indicate direcⁿ of thrust component on driving gear.

— for driven opposite to that for driving gear.

Example ②:- The layout of a double-reaction helical gearbox is shown in fig. Pinion A is the driving gear and 10 kW power at 720 rpm is supplied to it through its shaft no. 1. The no. of teeth on different helical gears are as follows:-

$Z_A = 20$, $Z_B = 50$, $Z_C = 20$, $Z_D = 60$.



The normal pressure angle for all gears is 20°. For the pair of helical gears A and B, the helical angle is 30° and normal module is 3 mm. For the pair C and D, the helix angle is 25° and the normal module is 5 mm. Pinion A has right handed helical teeth, while the pinion C has left-handed helical teeth. The bearings B1 & B2 are mounted on shaft no. 2 in such a way that Bearings B1 can take only radial load, while the bearing B2 can take both radial as well as thrust load. Determine the magnitude and direction of bearing reactions on shaft no. 2.

— $d_A = \frac{Z\, m_n}{\cos \psi}$

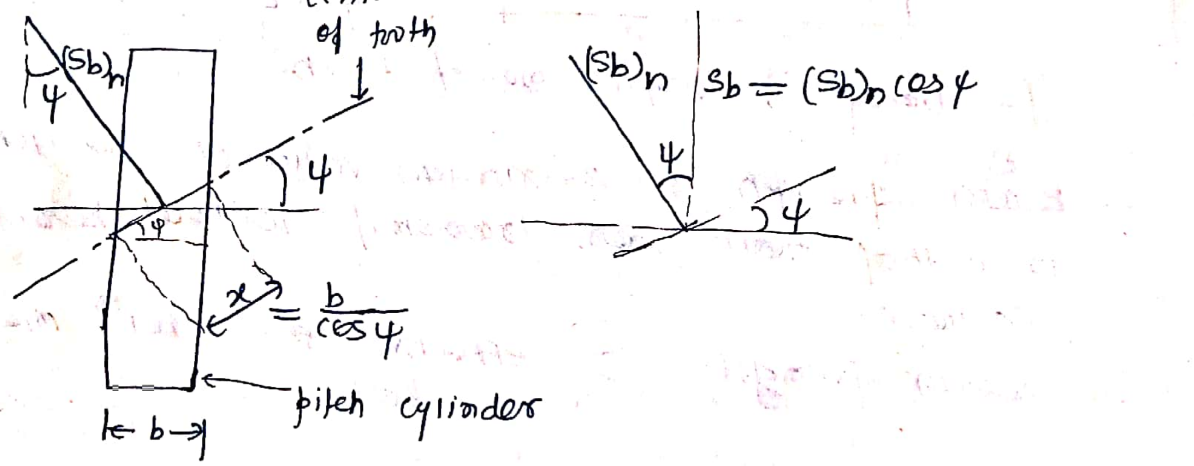
Beam strength of Helical Gear :-

- In order to determine beam strength, the helical gear is considered to be equivalent to a formative spur gear.
- The formative spur gear is imaginary spur gear in a plane perpendicular to the tooth element
- the pitch circle dia. of formative spur gear d', no. of teeth z' and module m_n.

Beam strength equation for spur gear is

$$S_b = m \cdot b \cdot \sigma_b \cdot Y \quad \text{———} \quad (1)$$

The above eqn is also applicable for formative spur gear



Equation of beam strength for formative gear is

$$(S_b)_n = m_n \cdot x \cdot \sigma_b \cdot Y \quad \text{———} \quad (2)$$

$\leftarrow$ depend on no. of teeth of formative gear z'

$$z' = \frac{z}{\cos^3\psi}$$

$(S_b)_n$ = beam strength perpendicular to tooth element

m_n = normal module (formative gear module in normal plane)

x = tooth width in normal plane $= \frac{b}{\cos\psi}$

Y = lewis form factor based on virtual no. of teeth $z' = \frac{z}{\cos^3\psi}$

In figure (S_b) is the component of $(S_b)_n$ in plane of rotation.

$$(S_b) = (S_b)_n \cos\psi$$

$$(S_b)_n = (S_b / \cos\psi) \quad — ③$$

$$a = b/\cos\psi$$

substituting ③ in ②

$$\frac{S_b}{\cos\psi} = m_n \cdot \frac{b}{\cos\psi} \cdot \sigma_b \cdot Y$$

$$\boxed{S_b = m_n \cdot b \cdot \sigma_b \cdot Y} \rightarrow \text{Lewis eq}^n \text{ for Helical gear}$$

Y - based on virtual no. of teeth.

- Beam strength is maximum value of tangential force that tooth can transmit without bending failure.
- Beam strength > effective force bet^n meshing teeth.

* Effective load on Gear :-

$$M_t = \frac{60\times10^6 (P_{kW})}{2\pi n}$$

$$P_t = \frac{2M_t}{d} \longrightarrow d = \frac{m_n \cdot z}{\cos\psi}$$

- The above eqn of tangential component depends on rated power and rated torque. In addition to that there is dynamic load

- Dynamic load - Approximate estimation by means of velocity factor in preliminary stages of gear design.

$$P_{eff} = \frac{C_s}{C_v} \cdot P_t$$

C_s = service factor
C_v = velocity factor

$$C_v = \frac{5.6}{5.6+\sqrt{v}}$$

v = pitch line velocity in m/s

$$= \frac{\pi d n}{60\times10^3} \text{ m/s}$$

(d ← mm, n ← rpm)

- Dynamic load - precise calculation by Buckingham's equation in final stage.

when gear dimension are known, error specified & quality of gear is determined.

$$P_{eff} = C_s \cdot P_t + P_d$$

$$P_d = \frac{21V(Ceb\cdot\cos^2\psi + P_t)\cos\psi}{21V + \sqrt{Ceb\cos^2\psi + P_t}}$$

C = deformation factor N/mm²
e = sum of error betn two meshing teeth (mm)
V = pitch line velocity m/s
ψ = helix angle.

In order to avoid failure of gear teeth due to bending

$$S_b > P_{eff}.$$

$$S_b = P_{eff} \cdot FS.$$

* Wear strength of Helical Gear :-

- Wear strength of spur gear is modified to helical gear
- a pair of helical gear is considered to be equivalent to a formative pinion and formative gear in a plane perpendicular to the tooth element.

- Wear strength of spur gear is

$$S_w = b Q d_p' K \quad \text{——— (1)}$$

- for a formative gear

$S_w = (S_w)_n$ = wear strength perpendicular to tooth element

$d_p' = \dfrac{d_p}{\cos^2\psi}$ = pitch circle dia. of formative pinion

width $= \dfrac{b}{\cos\psi}$

substituting it in (1)

$$(S_w)_n = \frac{b}{\cos\psi} \cdot Q \cdot \frac{d_p}{\cos^2\psi} \cdot K$$

The component in the plane of rotation is denoted by S_w.

$$S_w = (S_w)_n \cos\psi \qquad (S_w)_n = \left(\frac{S_w}{\cos\psi}\right)$$

$$\frac{S_w}{\cancel{\cos\psi}} = \frac{b}{\cancel{\cos\psi}} \cdot Q \cdot \frac{d_p}{\cos^2\psi} \cdot K$$

$$\boxed{S_w = \frac{b\, Q\, d_p \cdot K}{\cos^2\psi}}$$ → Buckingham's eqn of wear strength.

- Wear strength is maximum tangential force that the tooth can transmit without pitting failure.

$S_w > P_{eff}$ → To avoid pitting failure

- virtual no. of teeth on pinion & gear Z_p' & Z_g'

Ratio factor for external helical gear

$$\varphi = \frac{2 Z_g'}{Z_g' + Z_p'} \qquad \text{But } Z_g' = \frac{Z_g}{\cos^3\psi} \qquad Z_p' = \frac{Z_p}{\cos^3\psi}$$

$$\left[\varphi = \frac{2 Z_g}{Z_g + Z_p}\right]$$

Ratio factor internal helical gear

$$\left[\varphi = \frac{2 Z_g}{Z_g - Z_p}\right]$$

Z_g & Z_p = actual no. of teeth on gear & pinion resp.t.

— Pressure angle in plane perpendicular to tooth element (α_n)

$$K = \frac{\sigma_c^2 \cdot \sin\alpha_n \cos\alpha_n \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}$$

σ_c = endurance surface strength. (N/mm^2)

α_n = normal press angle ($20°$)

$$K = 0.16 \left(\frac{BHN}{100}\right)^2 \rightarrow \text{steel } \& \ 20° = \alpha_n.$$

- In order to avoid failure of gear tooth due to pitting

$$S_w > P_{eff}.$$

$$S_w = P_{eff}(F.S)$$

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Example :-

① A pair of parallel helical gears consists of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 rpm. The normal pressure angle is 20°, while the helix angle is 25°. The face width is 40 mm and normal module is 4 mm. The pinion as well as the gear is made of steel 40C8 ($S_{ut} = 600\ N/mm^2$) and heat treated to a surface hardness of 300 BHN. The service factor of safety are 1.5 & 2 respt. Assume that the velocity factor accounts for dynamic load and calculate the power transmitting capacity of gears.

→ Given :-

$n_p = 720$ rpm

$Z_p = 20$

$Z_g = 100$

$m_n = 4$ mm

$b = 40$ mm

$\psi = 25°$

$\alpha_n = 20°$

$S_{ut} = 600\ N/mm^2$

BHN = 3000

$C_s = 1.5$

FS = 2

- Both gears are made of same material so ~~gears~~ pinion is weaker.

- In which strength ~~is~~ pinion is weaker, in order to find criteria of design, find S_b & S_w.

Beam strength. $S_b = m_n b \cdot \sigma_b Y$

$$Z'_p = \frac{Z_p}{\cos^3\psi} = \frac{20}{\cos^3 25} = 26.87$$

$$Y = 0.344 + \frac{(0.348-0.344)(26.87-26)}{27-26} = 0.3475$$

$$\sigma_b = \frac{S_{ut}}{3} = 200\ N/mm^2$$

$$S_b = m_n b \sigma_b Y = 4 \times 40 \times 200 \times 0.3475 = 11120\ N.$$

Wear strength :-

$$Q = \frac{2Z_g}{Z_g + Z_p} = 1.667.$$

$$d_p = \frac{Z_p \cdot m_n}{\cos \psi} = 88.27 \text{ mm}$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2 = 1.44 \text{ N/mm}^2$$

$$S_w = \frac{bQd_pK}{\cos^2\psi} = \frac{40 \times 1.667 \times 88.27 \times 1.44}{\cos^2 25} = 10318.58 \text{ N}$$

since wear strength is lower than beam strength, pitting is the criteria of failure

$$v = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi (88.27)(720)}{60 \times 10^3} = 3.328 \text{ m/s}$$

$$C_v = \frac{5.6}{5.6 + \sqrt{v}} = 0.7543$$

$$S_w = \frac{C_s}{C_v} P_t \cdot (FS)$$

$$10318.58 = \frac{1.5 \times 2 \times P_t}{0.7543}$$

$$\boxed{P_t = 2594.43 \text{ N}}$$

$$M_t = \frac{P_t d_p}{2} = \frac{2594.43 \times 88.27}{2} = 114505.39 \text{ Nmm}$$

$$P_{kw} = \frac{2\pi n_p M_t}{60 \times 10^6} = 8.63 \text{ kW}$$

Example

(2) A pair of parallel helical gears consists of 24 teeth pinion rotating at 5000 rpm and supplying 2.5 kW power to a gear. The speed reduction is 4:1. The normal pressure angle and helix angle are 20° & 23° respectively. Both gears are made of hardened steel (Sut = 750 N/mm²) The service factor and the factor of safety are 1.5 & 2 respectively. The gears are finished to meet the accuracy of grade 4.

i) In the initial stage of gear design, assume that the velocity factor accounts for the dynamic load and that the face width is 10 times the normal module. Assuming the pitch line velocity to be 10 m/s. Estimate the normal module.

ii) Select the first preference value of the normal module and calculate the main dimensions of the gears

iii) Determine the dynamic load using Buckingham's eqn and find out the effective load for the above dimensions. What is the correct factor of safety for bending ?

iv) Specify surface hardness for the gears assuming a factor of safety of 2 for wear consideration.

$$M_t = \frac{60 \times 10^6 \, P_{kW}}{2\pi n_p} = 4774.648 \text{ Nmm}$$

$$d_p = \frac{Z_p m_n}{\cos \psi} = \frac{24 m_n}{\cos 23} = 26.073 m_n \text{ mm}$$

$$P_t = \frac{2M_t}{d_p} = \frac{366.25}{m_n} \text{ N}$$

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$$C_v = \frac{5.6}{5.6+\sqrt{v}} = 0.6391$$

$$P_{eff} = \frac{C_s}{C_v} P_t = \frac{366.25}{m_n}\left(\frac{1.5}{0.6391}\right) = \frac{859.61}{m_n} N.$$

$$Z_p' = \frac{Z_p}{\cos^3\psi} = 30.77.$$

$$Y = 0.358 + \frac{(0.364-0.358)(30.77-30)}{(32-30)} = 0.36$$

$$\sigma_b = \frac{S_{ut}}{3} = 250\ N/mm^2$$

$$S_b = m_n b \sigma_b Y = m_n (10 m_n)(250)(0.36) = 900\, m_n^2\ N.$$

$$S_b = P_{eff}\ fs$$

$$\boxed{m_n = 1.24\ mm} \simeq 1.5\ mm$$

Gear Dimensions $m_n = 1.5\ mm$

$b = 10 m_n = 15\ mm$, $d_p = \frac{Z_p m_n}{\cos\psi} = 39.11$, $d_g = \frac{Z_g m_n}{\cos\psi} = 156.44\ mm$

Beam strength

$$S_b = m_n b \sigma_b Y = 2025\ N$$

$$P_t = \frac{2M_t}{d_p} = 244.16\ N$$

Dynamic load :- Grade 4 $\quad e = 3.2 + 0.25\phi$

$\phi = m_n + 0.25$

for pinion

$$e_p = 3.2 + 0.25(1.5 + 0.25\sqrt{39.11}) = 3.9659\ \mu m$$

Gear $e_g = 3.2 + 0.25(1.5 + 0.25\sqrt{156.44}) = 4.3567\ \mu m$

$$e = 3.9659 + 4.3569 = 8.3226 \times 10^{-3}\ mm$$

$C = 11400\ N/mm^2 \qquad b = 15\ mm.$

$$V = \frac{\pi d_p n_p}{60 \times 10^3} = 10.24 \text{ m/s}$$

$$P_d = \frac{21V\left[\left(C e_b \cos^2\psi + P_t\right)\cos\psi\right]}{21V + \sqrt{C e_b \cos^2\psi + P_t}}$$

$$= 1133.94 \text{ N.}$$

$$P_{eff} = C_s P_t + P_d = 1500.18 \text{ N}$$

$$f_s = \frac{S_b}{P_{eff}} = 1.35$$

Surface Hardness :-

$$S_w = P_{eff} \cdot f_s = 1500.18 \times 2 = 3000.36 \text{ N.}$$

$$Q = \frac{2 Z_g}{Z_g + Z_p} = \frac{2 \times 96}{96 + 24} = 1.6.$$

$$S_w = \frac{b Q d_p K}{\cos^2\psi.}$$

$$3000.36 = \frac{15 \times 1.6 \times 39.11\left[0.16\left(\frac{BHN}{100}\right)^2\right]}{\cos^2(23)}$$

$$\boxed{BHN = 411.24 \simeq 420}$$

Example:-

The following data is given for a steel helical gear pair transmitting 150 kW power from a shaft rotating at 1440 rpm to another parallel shaft rotating at 360 rpm.

centre distance = approx. 435 mm

Helical angle = 24°.

face width = 14 mn.

Number of teeth on pinion = 20

Permissible bending stress for pinion material = 152 N/mm²

—— " —— gear material = 125 N/mm²

tooth system = 20° full Depth involute.

service factor = 1.53

combined teeth error = 0.0406 mm

Deformation factor = 11600 e N/mm

Assuming the dynamic load is accounted by the Buckingham's equation calculate

i) the factor of safety against bending failure &

ii) surface hardness, if the FOS against pitting failure is 1.5